

Open Problems in Sturm-Liouville Theory

(Angelo B. Mingarelli¹)

$$\begin{aligned} -(p(x)y')' + q(x)y &= \lambda r(x)y, & a \leq x \leq b, \\ y(a) &= y(b) = 0. \end{aligned}$$

Simplest case: p, q, r real, piecewise continuous over I . Eigenvalue problem is **NON-DEFINITE** if for every $\alpha \in \mathbb{R}$, the form

$$\int_a^b (p|y'|^2 + (q - \alpha r)|y|^2) dx$$

is **indefinite** on a suitable space of test functions.

FACTS: The eigenvalue problem may have **non-real eigenvalues** (Conjectured by R.G.D.Richardson (1918); resolved by ABM (1982)); the non-real spectrum may **fill the whole complex plane** (Atkinson-Mingarelli, Crelle's J. (1983)); non-real eigenvalues preclude the existence of **ground states** (Allegretto-Mingarelli, Crelle's J. (1989)). Survey paper preprint (1900-1986) can be downloaded from: <http://arxiv.org/pdf/1106.6013.pdf>

Let $p = 1$.

QUESTION 1: Find sufficient conditions depending on q, r , for the existence of at least one non-real eigenvalue.

QUESTION 2: How many non-real eigenvalues, if any, does a given problem have?

QUESTION 3: Is there a general oscillation theorem for the real and imaginary parts of non-real eigenfunctions?

More than 40 open questions can be found in:

F. V. Atkinson, A. B. Mingarelli, *Multiparameter Eigenvalue Problems* (Sturm-Liouville Theory), CRC Press, Taylor and Francis, (2010). ISBN 13:9781439816226

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